

Sensitivity of attenuation length measurements

This document discusses the relation between the sensitivity of attenuation measurements made on soak samples in the mine to the size of possible effects in the real KamLAND detector.

I conclude that the test cell measurements have more than enough sensitivity to satisfy us that tested materials will not affect the light transmission in the detector either for physics events in the full volume or calibration events localized near the possible contamination.

1 Formalism

If a sample of liquid scintillator with attenuation length Λ_{clean} is contaminated with a fraction x of a contaminant with attenuation length Λ_{cont} , the attenuation length of the contaminated scintillator Λ_{dirty} related to the original attenuation length by:

$$\frac{1}{\Lambda_{\text{dirty}}} = \frac{1-x}{\Lambda_{\text{clean}}} + \frac{x}{\Lambda_{\text{cont}}}. \quad (1)$$

The ratio R between dirty and clean scintillator of the light transmitted through distance d is:

$$R = \frac{e^{-d/\Lambda_{\text{dirty}}}}{e^{-d/\Lambda_{\text{clean}}}} = e^{-d\left(\frac{1}{\Lambda_{\text{dirty}}} - \frac{1}{\Lambda_{\text{clean}}}\right)} = e^{-xd\left(\frac{1}{\Lambda_{\text{cont}}} - \frac{1}{\Lambda_{\text{clean}}}\right)}. \quad (2)$$

As we are dealing with small concentrations of contaminants, $x \ll 1$, we can approximate the ratio as:

$$R = 1 + xd\left(\frac{1}{\Lambda_{\text{cont}}} - \frac{1}{\Lambda_{\text{clean}}}\right). \quad (3)$$

The fractional change in light output, $\frac{\Delta L}{L}$ is just

$$\frac{\Delta L}{L} = 1 - R = xd\left(\frac{1}{\Lambda_{\text{clean}}} - \frac{1}{\Lambda_{\text{cont}}}\right). \quad (4)$$

1.1 Test cell measurement

The attenuation length measurement in the mine compares two samples, one of clean and one of dirty scintillator. This differential measurement is sensitive to the fractional change in light transmission over the size of the test cell, a distance $d_0 = 10$ cm.

$$\left(\frac{\Delta L}{L}\right)_0 = d_0 x_0 \left(\frac{1}{\Lambda_{\text{clean}}} - \frac{1}{\Lambda_{\text{cont}}}\right). \quad (5)$$

The sensitivity of the measurement apparatus in the mine is systematically limited to 2% [private conversations with Kengo, based on his conversations with Suekane]. In other words, we cannot measure $\left(\frac{\Delta L}{L}\right)_0 < 2\%$.

1.2 Simple cases

We want to relate the light attenuation in the test cell to changes in reconstructed energy in the detector. In the discussion below, I assume reconstructed energy is proportional to transmitted light, and I neglect reemission.

1.2.1 Uniform contamination

If we assume the contamination is distributed uniformly through the balloon, we can calculate the change in reconstructed energy for events at the center of the detector. Light deposited in the center travels through $d_1 = 6.5$ m of scintillator en route to the PMTs. For a diluted concentration x_1 , the change in reconstructed energy is:

$$\left(\frac{\Delta E}{E}\right)_1 = d_1 x_1 \left(\frac{1}{\Lambda_{\text{clean}}} - \frac{1}{\Lambda_{\text{cont}}}\right), \quad (6)$$

$$= \frac{d_1 x_1}{d_0 x_0} \left(\frac{\Delta L}{L}\right)_0. \quad (7)$$

A typical soak sample uses 50 g of scintillator, whereas we have 1 kton of scintillator in the 6.5 m balloon. If the total amount of contaminant in the soak sample were distributed through the whole scintillator volume, the contamination would be $x_1 = 5 \times 10^{-8} x_0$. The ratio of the 6.5 m light path to the 10 cm test cell is $d_1 = 65 d_0$. We therefore find:

$$\left(\frac{\Delta E}{E}\right)_1 = 3.25 \times 10^{-6} \left(\frac{\Delta L}{L}\right)_0. \quad (8)$$

The 2% measurement sensitivity of the attenuation length measurement corresponds to a 6.5×10^{-8} change in reconstructed energy for the case of uniform concentration and events at the center of the detector.

1.2.2 Localized contamination, 30 cm

The more troublesome case is contamination that remains localized in the detector. Suppose that instead of dispersing through the entire detector, the contamination remains in a volume of radius $x_2 = 30$ cm, $x_2 = 3x_1$ and it only affects the scintillator optical properties in that volume. For simplicity, if we assume the concentration to be constant, I find $x_2 = 5.1 \times 10^{-4}x_0$. In this case, for light produced in the center of the contaminated region,

$$\left(\frac{\Delta E}{E}\right)_1 = 1.53 \times 10^{-3} \left(\frac{\Delta L}{L}\right)_0. \quad (9)$$

The 2% measurement sensitivity of the attenuation length measurement corresponds to a 3.1×10^{-5} change in reconstructed energy for this case of localized concentration in a 30 cm radius.

1.3 General localized contamination

More generally, in the limit of small contamination, the effect of contaminating for events at the center of the contaminated region goes linearly with the size of the region, and linearly with the concentration of contaminant. Since for a given amount of contaminant, the concentration goes as the inverse cube of the size of the contaminated region, the total effect goes as the inverse square of the size of the region. Taking full-volume contamination as a reference point:

$$\left(\frac{\Delta E}{E}\right)_3 = 3.25 \times 10^{-6} \left(\frac{d_3}{6.5\text{ m}}\right)^{-2} \left(\frac{\Delta L}{L}\right)_0 \quad (10)$$

The 2% sensitivity of the attenuation length measurement corresponds to a 0.1% change in reconstructed energy for contamination localized to a 5.2 cm sphere around the light emission.

Intuitively, one might expect that a 2% light attenuation in the 10 cm test cell should correspond to a 2% effect on reconstructed energy if the contamination is limited to a 10 cm region around an event. On the other hand, the calculation above gives a 2.7×10^{-4} effect, a factor of 1.4×10^{-2} smaller. Note, however, that in the test cell measurement the contaminant is dissolved in 50 g of scintillator, whereas a 10 cm sphere of scintillator is

3.6 kg of scintillator, in which is same amount of contaminant would be 1.4×10^{-2} times more dilute.

2 Discussion

If a small amount of contaminant is dispersed through all the liquid scintillator, the volume is so large that we could not possibly see any effect on general physics events. Even for localized contamination, the overall effect on physics data will be tiny, because very little of the overall light output of events from the entire detector will pass through a localized area of contamination.

The main concern is really how optical contamination might effect calibration data. If contaminants remain localized in the detector, they will be near the calibration sources, and our calibrations could therefore all be biased relative to the bulk detector properties.

However, the distance scale of contaminant localization needed to produce even very small effects is extremely short. The 2% sensitivity of the test setup would produce a 0.1% effect only for events at the center of a 5.1 cm-radius sphere of contamination. It is implausible that contaminants would remain confined to such a small volume. In fact, contaminant localization on this small length scale would call into question our entire measurement approach, which assumes that the contaminants are uniform over a 10 cm test cell. Furthermore, the gammas from typical calibration sources travel well outside a 5 cm radius around the source before interacting.

To put the relation between the test cell measurement and possible effects in the detector on completely firm quantitative ground, we would need a physical understanding of how far a contaminant would actually diffuse in the scintillator.

However, even in the absence of this exact knowledge, it is clear that the test cell measurements are at least two orders of magnitude more sensitive than largest conceivable effects on calibration sources in the KamLAND detector.